Zur Photometrie zerstreut reflectirender Substanzen.
Von H. Seeliger.

Sitzungsberichte
der
mathematisch-physikalischen Classe
der
k. b. Akademie der Wissenschaften
zu München.
Band XVIII. Jahrgang 1888.
München
1889.

**1888**
(Meeting report of the scientific academy of Bavaria)

gestellten Betrachtungen zu knüpfen, scheint mir nicht am Platze. Die von Lommel[1]) zuerst consequent durchgeführte Absorptionstheorie führt auf die Formel

Lommel-Seeliger law
$$q = \gamma \cdot \frac{\cos i \cos \varepsilon}{k \cos i + \cos \varepsilon} \quad (3)$$



## Single+multiple scattering in a semi-infinite atmosphere
## Hapke's 1981 breakthrough

Lommel-Seeliger law; Chandrasekhar's classic solution (H-functions)

$$I_0 = \frac{\omega I_\star}{4} \frac{\mu_\star}{\mu_\star + \mu} \left( P_\star + HH_\star - 1 \right)$$

single scattering

Hapke's solution:
$$H(\mu) = \frac{1+2\mu}{1+2\gamma\mu}$$

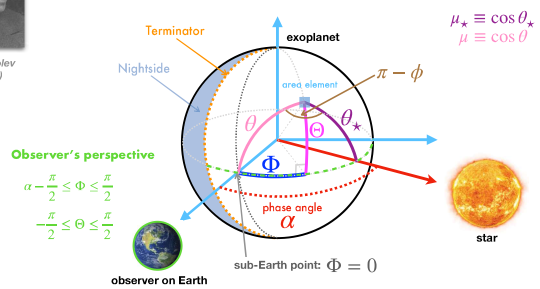

single scattering


multiple scattering (isotropic)




Viktor V. Sobolev (1915-1999)

## Problem may be visualised in two coordinate systems: observer-centric versus local




$$\mu = \cos\Theta \ \cos\Phi,$$
$$\mu_\star = \cos\Theta \ \cos(\alpha - \Phi),$$
$$\cos\alpha = \mu\mu_\star - \sqrt{\left(1-\mu^2\right)\left(1-\mu_\star^2\right)} \ \cos\phi .$$

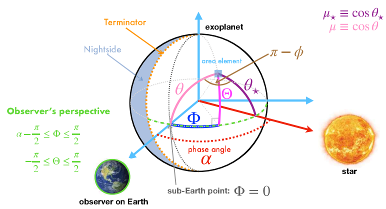
**Key point:**
Depending on which quantity you are trying to derive, relationships between angles may be messy in one coordinate system, but "clean" in another!



## Key insights

1. In observer-centric coordinate system, three angles are independent:

$$\Theta, \ \Phi, \ \alpha$$



2. Scattering and orbital phase angles are trivially related:

$$\beta = \pi - \alpha$$

3. Broad class of reflection laws depend only on scattering angle:

$$P(\beta) \text{ only}$$

Insights imply that one may derive integral phase function for *any* reflection law, if one works in correct coordinate system!



Heng, Morris & Kitzmann (2021, Nature Astronomy, 5, 1001)


Viktor V. Sobolev (1915-1999)

## Sobolev's insight: the reflection coefficient

$$\rho = \frac{I_0}{\mu_\star I_\star}$$
($I_0$: emergent intensity; $I_\star$: incident intensity)

Key quantities of interest can be expressed as *integrals* involving the reflection coefficient

$$A_g = 2 \int_0^1 \rho_0 \ \mu^2 \ d\mu$$

The subscript "0" means the quantity is evaluated at zero orbital phase angle.

$$F = \int_{\alpha-\pi/2}^{\pi/2} \int_0^{\pi/2} \rho\mu\mu_\star \cos\Theta \ d\Theta \ d\Phi \implies \Psi = \frac{F}{F_0}$$

Sobolev flux



## Sobolev's insight: the reflection coefficient

$$\rho = \frac{I_0}{\mu_\star I_\star}$$
($I_0$: emergent intensity; $I_\star$: incident intensity)

Note the difference between isotropic and Lambertian:

$$\rho = \frac{\omega}{4} \frac{HH_\star}{\mu_\star + \mu} \quad \text{vs} \quad \rho = 1$$