THE ASTROPHYSICAL JOURNAL, 538:885–903, 2000 August 1

ALBEDO AND REFLECTION SPECTRA OF EXTRASOLAR GIANT PLANETS

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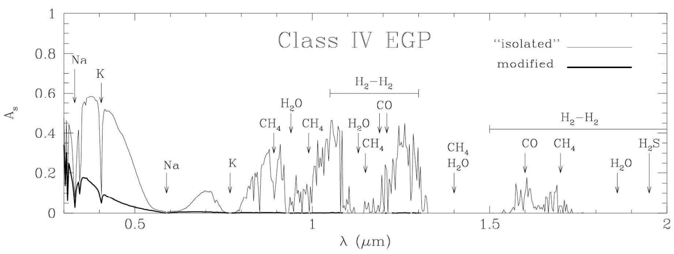
Received 1999 October 27; accepted 2000 March 13

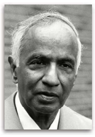


This paper already predicted that hot Jupiters will be "dark" (~10% albedo in the visible)

Historical background & theory

Johann Heinrich Lambert (1728-1777)

- Lambert's law of reflection
- "Lambertian sphere" model



Subrahmanyan Chandrasekhar (1910-1995)

- Exact solution for isotropic scattering in semi-infinite atmosphere

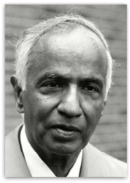
Viktor V. Sobolev (1915-1999)

- Russian master of radiative transfer
- Influential 1975 textbook

Bruce Hapke (1931-present)

- Hapke (1981): key approximation to Chandrasekhar's solution

Chandrasekhar's seminal contributions to radiative transfer theory



Subrahmanyan Chandrasekhar (1910-1995)
Indian-American Nobel laureate (1983)

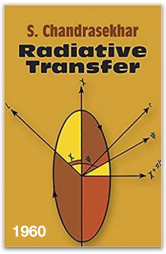
Radiative transfer equation may be solved exactly in the limit of a **semi-infinite atmosphere** with **isotropic scattering**

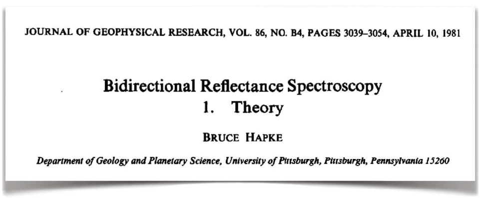
Solution is expressed implicitly in terms of **Chandrasekhar *H*-functions**:

$$H(\mu) = 1 + \tfrac{1}{2} w \mu H(\mu) \int_0^1 \frac{H(\mu')}{\mu + \mu'} d\mu'$$

Requires numerical iteration!

Exists a literature on *H*-functions alone




JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 86, NO. B4, PAGES 3039–3054, APRIL 10, 1981

Bidirectional Reflectance Spectroscopy
1. Theory

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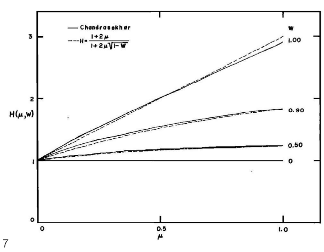
Bruce Hapke (1931-present)

Hapke (1981) discovered an approximate, but highly accurate, closed-form solution to the Chandrasekhar *H*-function:

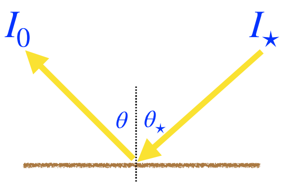
$$H(\mu) = 1 + \tfrac{1}{2} w \mu H(\mu) \int_0^1 \frac{H(\mu')}{\mu + \mu'} d\mu'$$

$$H(\mu) = \frac{1 + 2\mu}{1 + 2\gamma\mu}$$

$\gamma = \sqrt{1 - \omega}$



Single scattering in a semi-infinite atmosphere
Lommel-Seeliger Law



$\mu_\star = \cos\theta_\star$

$\mu = \cos\theta$

single-scattering albedo

$$I_0 = \frac{\omega I_\star}{4} \frac{\mu_\star}{\mu_\star + \mu} P_\star$$

reflection law (scattering phase function)

One can *derive* this expression from the radiative transfer equation in the limit of a semi-infinite atmosphere