How to make a model?

- Modeling the planetary evolution.
- Energy balance equation: $L = 4\pi\sigma R^2 T_{\rm eff}^4$

$$L = L_{\odot} + L_{\rm int}$$

- Need an atmospheric model to relates $T_{\rm eff}$ to a physical temperature like $T_{\rm 1bar}$.
- Starting from an initial model, one can estimate the timestep corresponding to a specific entropy difference.

Equations of state (EOS)

- The EOS tells how material properties depend on the environmental conditions.
- We need rho(P, T) to solve the set of equations. Also entropy, thermal and electrical conductivities, …

$$\rho = \rho(P, T, X_i) \qquad S = S(P, T, X_i)$$

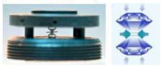
- High-pressure experiments and theoretical calculations are conducted.

More references: *McMahon et al.* 2012

Equations of state (EOS)

- Experiments:

Diamond anvil cells ~ a few Mbar

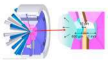


Loubeyre et al. 2012
Zaghoo et al. 2016

Shock experiments 5 Mbar

Gas-gun

Omega laser (LLE, Rochester)



Knudson & Desjarlais 2017
Brygoo et al. 2021

Equations of state (EOS)

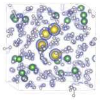
- Theoretical calculations

Chemical picture

Modeling the interactions between atoms and molecules using pair-potentials.

Physical picture

Density Functional Theory (DFT), Quantum Monte Carlo (QMC)



Soubiran & Militzer 2012

Equations of state (EOS)

- Pros and cons

Experiments

✓ Direct measurements. More trustworthy?

✗ Cost a lot. Limited range of P, T conditions.

Theoretical calculations

✓ Larger range of P, T conditions.

✗ Computationally expensive. Still rely on some approximations (XC-functionals).

- Planetary scientists should keep working in collaboration with high-pressure physicists!

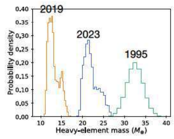
Equations of state (EOS)

- Why is it so important?

Jupiter's first interior model in 1924. Two things led to major improvements: better gravity data and better knowledge of the behavior of Hydrogen at high pressures!

- An example of how the inferred composition can be affected by the EOS.

Jupiter (318 $M_{\rm earth}$) and the H-He EOS



Equations of state (EOS)

- For which elements?

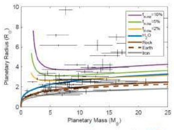
Hydrogen is crucial. It dominates gas giants.

Helium is also important.

Ices and **rocks**? Yes and no.

Given the uncertainties on radius/mass observations for exoplanets (and the degeneracy of solutions), accurate EOSs are less needed. A minor change of the amount of gas or changing types of ices or rocks can easily correct for substantial changes in the density profiles.

Keep in mind that EOSs remain uncertain although we've done a lot of progress.



Lozovsky et al. 2019

Equations of state (EOS)

- Mixtures

In planets, most of the times, you have mixtures, not single layers made of one single element. Pure element EOS are only the end members.

- How to model? **Linear mixing** or **Additive volume law** (Recipe to combine different elements)

Numerical simulations with different atoms/molecules are more computationally expensive.

For W an extensive variable: $W(P,T) = \sum_i X_i W_i(P,T)$ $\quad X_i = \dfrac{M_i}{(\sum_i M_i)}$ Mass fractions

In the case of density: $\dfrac{1}{\rho_{\rm H-He-Z}} = \dfrac{X}{\rho_{\rm H}} + \dfrac{Y}{\rho_{\rm He}} + \dfrac{Z}{\rho_{\rm Z}}$

This is an approximation!

✓ tested with ternary mixtures (including gas, ices, rocks). *Soubiran & Militzer 2016*

✗ non-ideal effects on density for H-He interactions can reach up to ~10% *Howard & Guillot 2023*

Heat transport

- Heat is transported from the deep interior towards the surface.

$$\boxed{\frac{\partial T}{\partial r} = \frac{\partial P}{\partial r}\frac{T}{P}\nabla_T} \qquad \nabla_T = \frac{d\ln T}{d\ln P}$$

- Radiation and conduction can be handled in a similar way, both being treated together using:

$$1/\kappa = 1/\kappa_{\rm rad} + 1/\kappa_{\rm cond}$$

$\nabla_{\rm rad} \propto \dfrac{\kappa P L}{m T^4}$ Depends on opacities!

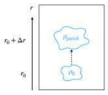
- The adiabatic gradient (in a convective medium) is defined by the EOS.

$\nabla_{\rm ad} = \left(\dfrac{d\ln T}{d\ln P}\right)_S \sim 0.3$ (How T changes with P at constant S)

- Comparing these gradients can tell what the driving mechanism is.

Heat transport

- To derive a criterion for convective instability, we do a thought experiment of a raising parcel of fluid.



We assume that it's fast enough so that it does not exchange heat with its surroundings. It's an **adiabatic** process.

The difference in density between the parcel and its surroundings will be:

$$D\rho = \left[\left(\frac{d\rho}{dr}\right)_{\rm parcel} - \left(\frac{d\rho}{dr}\right)_{\rm surr}\right]\Delta r,$$

If the parcel continues to rise, its unstable. If it sinks back, it is stable.

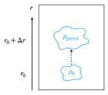
Hence, the stability criterion is simply: $\left(\frac{d\rho}{dr}\right)_{\rm parcel} > \left(\frac{d\rho}{dr}\right)_{\rm surr}$ (Impractical because as a function of density gradients)

We switch to temperatures using: $\frac{d\rho}{\rho} = \alpha\frac{dP}{P} - \delta\frac{dT}{T} + \phi\frac{d\mu}{\mu},$

$$\alpha = \left(\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln P}\right), \quad \delta = -\left(\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln T}\right), \quad \phi = \left(\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln\mu}\right).$$

Heat transport

- To derive a criterion for convective instability, we do a thought experiment of a raising parcel of fluid.



$$\delta\frac{P}{T}\left(\frac{dT}{dP}\right)_{\rm parcel} > \delta\frac{P}{T}\left(\frac{dT}{dP}\right)_{\rm surr} - \phi\frac{P}{\mu}\left(\frac{d\mu}{dP}\right)_{\rm surr}$$

Stable against convection if: $\boxed{\nabla_{\rm rad} < \nabla_{\rm ad} + \frac{\phi}{\delta}\nabla_{\mu},}$ **Ledoux criterion.**

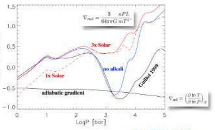
In case no composition gradient, Schwarzchild criterion: $\nabla_{\rm rad} < \nabla_{\rm ad}$

- Particular cases when you satisfy the Ledoux criterion but not the Schwarzchild criterion, e.g. double diffusive convection.

Heat transport

- Is the environment radiative or convective ?

Convective instability occurs when: $\nabla_{\rm rad} > \nabla_{\rm ad} + \frac{\phi}{\delta}\nabla_{\mu}$



Guillot et al. 2004

- Opacities from CH_4, H_2O, collisions induced absorption of H_2 and He. Alkali metals are good opacity providers.
- Jupiter should be mostly convective
- Opacities are uncertain

How to make a model? Summary

$$\frac{\partial m}{\partial r} = 4\pi r^2\rho$$

$$\frac{\partial P}{\partial r} = -\rho g$$

EOS is needed to solve this set of equations (knowledge of the behavior of multi-component systems at high pressure)

$$\frac{\partial T}{\partial r} = \frac{\partial P}{\partial r}\frac{T}{P}\nabla_T$$

Need to know how heat is transported to estimate ∇_T

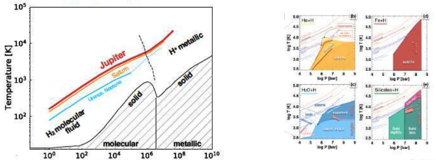
$$\frac{\partial L}{\partial r} = 4\pi r^2\rho\left(\dot{e} - T\frac{\partial S}{\partial t}\right)$$

At least 3 equations, 4 if you want to model the evolution.

Phase transitions / Phase separations

Thermodynamics is key!

- Phase transitions are everywhere.

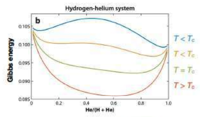


Guillot et al. 2023

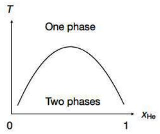
- It is important since it tells if a planet is layered or differentiated.
- Interior models need to assume a structure a priori (e.g. three-layer). Similarly as we needed some prior knowledge of the elements accreted by planets.

Phase transitions / Phase separations

- Phase transitions are everywhere.



Stevenson 2020



- Low temperatures favour immiscibility.
- Complexity increases when adding more elements.

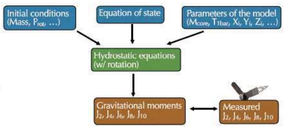
Outline

1. Why study interiors of planets?
2. What are the building blocks of planets?
3. **How to study planetary interiors?**
    - How to make a model?
    - **What observational constraints?**
4. What do models predict?
    - Results for Jupiter, Saturn, Uranus and Neptune
    - Results for exoplanets
5. Conclusion

© NASA/SwRI Kevin M. Gill

Inversion theory

- Inferences about the interiors are usually indirect.
- We look at something that emanates from the interior. That's why we need « inversion ». And inversion is non-unique.
- Typically, we calculate a model, compute its gravity field and compare it to the data.



Initial conditions (Mass, $P_{\rm rot}$, …)

Equation of state

Parameters of the model ($M_{\rm core}$, $T_{\rm 1bar}$, X_i, Y_i, Z_i, …)

Hydrostatic equations (w/ rotation)

Gravitational moments J_2, J_4, J_6, J_8, J_{10}

Measured J_2, J_4, J_6, J_8, J_{10}

- Optimization techniques, MCMC, ML…

What data?

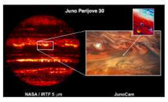
- Mass, radius.
- Rotation, winds.
- Luminosity.
- Atmospheric measurements (temperature, composition).
- Gravity field.
- Others: e.g. magnetic fields, seismology…

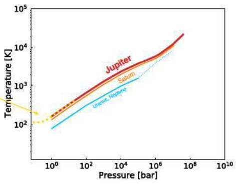
Atmospheric measurements

- Temperature

If the interior is fully convective, a measurement of T at 1 bar will constrain the whole temperature profile.

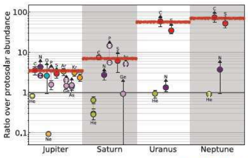
Jupiter's atmospheric temperature was measured by Galileo in 1995.





Atmospheric measurements

- Composition



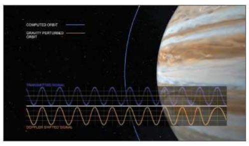
Guillot et al. 2023

- Isotopes too!

Gravity

- Because of their rotation, giant planets deviate from a perfectly spherical shape. The centrifugal force causes the planet to flatten at the poles and to bulge at the equator.

- This departure from sphericity affects the gravity field of the planet.
- Scrutinising the trajectory of a spacecraft around a planet and estimating the influence of the gravitational pull of this planet can tell about its density distribution.
- The so-called gravitational moments reflect the slightly perturbed spherical shape of the planet and are nowadays measured with high accuracy with flybys of spacecrafts.



Gravity

- Theory of figures by *Zharkov & Trubitsyn* 1978. Translated by W. B. Hubbard.
- Accounting for rotation in the hydrostatic equilibrium equation:

$$\frac{\nabla P}{\rho} = \nabla V + \nabla Q \qquad \text{with } V(r) = G\int \frac{\rho(r')}{|r-r'|}d\tau' \text{ the gravitational potential}$$

$$Q(r) = \frac{1}{2}\omega^2 r^2 \sin^2(\theta) \quad \text{the centrifugal potential}$$

- The gravitational potential can be decomposed in a series of Legendre polynomials:

$$\frac{1}{|r-r'|} = \begin{cases} \frac{1}{r}\sum_{n=0}^{\infty}(\frac{r'}{r})^n P_n(t), & \text{if } r > r' \text{ (external)}. \\ \frac{1}{r}\sum_{n=0}^{\infty}(\frac{r'}{r})^{-n-1} P_n(t), & \text{if } r < r' \text{ (internal)}. \end{cases}$$

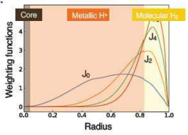
- And can therefore be written as: $V(r) = \frac{G}{r}\sum_{n=0}^{\infty} P_n(\cos\theta)\int \rho(r')P_n(\cos\theta')(r'/r)^k d\tau'.$

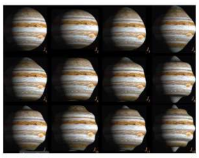
Gravity

- Assuming axisymmetry and symmetry between the northern and southern hemispheres the external gravitational potential can be written:

$$V_{ext} = \frac{GM}{R}\left[1 - \sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{2n} J_{2n}P_{2n}(cos\theta)\right] \quad \text{with} \quad J_{2n} = -\frac{1}{Ma^{2n}}\int \rho(r',\theta)(r')^{2n}P_{2n}(cos\theta)d^3r' \quad \text{the gravitational moments}$$

- The planetary density profile is constrained, not composition. The presence of a core is inferred indirectly.





Outline

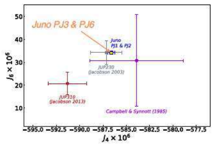
1. Why study interiors of planets?
2. What are the building blocks of planets?
3. How to study planetary interiors?
    - How to make a model?
    - What observational constraints?
4. **What do models predict?**
    - **Results for Jupiter, Saturn, Uranus and Neptune**
    - Results for exoplanets
5. Conclusion

© NASA/SWRI
Kevin M. Gill

Gravitational moments measurements for Jupiter

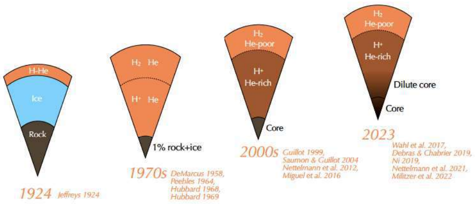
Jupiter was visited by:

- Pioneer 10 & 11 in the early 70s
- Voyager 1 & 2 in the late 70s
- Galileo in 1995
- Cassini and New Horizons in the 2000s
- Juno since 2016



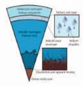
Iess et al. 2018

Jupiter's evolution in the last century

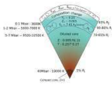


A dilute core inside Jupiter

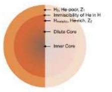
- Latest static models that fit the gravity data measured by Juno suggest the presence of a dilute core inside Jupiter, i.e. a region where the heavy elements are gradually mixed in the H-He envelope.



Wahl et al. 2017



Debras & Chabrier 2019



Miguel et al. 2022

- Jupiter's envelope is inhomogeneous. Important for formation models: how such a dilute core formed?
- Implications on the temperature structure. Non-adiabatic interior?
- Is there still a pure heavy-element (Z=1) core?