Sources

- Kippenhahn et al. 2013: Stellar structure and evolution
- Lecture notes from David Stevenson
- Review papers by Tristan Guillot, Ravit Helled, Jonathan Fortney, Nadine Nettelmann

Outline

1. Why study interiors of planets?
2. What are the building blocks of planets?
3. How to study planetary interiors?
   How to make a model?
   What observational constraints?
4. What do models predict?
   Results for Jupiter, Saturn, Uranus and Neptune
   Results for exoplanets
5. Conclusion

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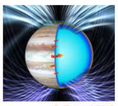
To understand many phenomena

- Why do some planets have magnetic fields?
  Metallic hydrogen in Jupiter.
  Ionic water in Uranus?

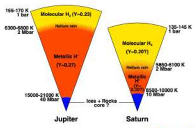


Guillot et al. 2018

- Why does a planet emit that amount of heat?
- Correctly interpreting atmospheric observations.
  Link between atmosphere and interior (e.g. helium rain).

A testing ground for fundamental physics

- Planetary interiors are natural laboratories.
  « Extreme » conditions:
  Pressure can reach dozens of Mbar.
  Temperatures up to several 10^4 K.

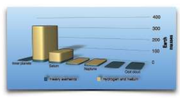


Guillot et al. 2005

- Metallic hydrogen was predicted in 1935 by Wigner and Huntington.
  Confirmed experimentally « recently ».
- Helium becomes immiscible with metallic hydrogen at sufficiently low temperatures.

To understand how planets form

- Planets are what they eat.
  Their present-day structure and composition are constraints for formation models. Reconciling formation/evolution/interior is key!
- Why giant planets?
  They are the first to form as they are made of primordial gas. They highly influence the history of the planetary systems.



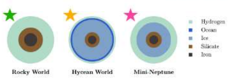
In our solar system, giant planets possess 99.5% of all the mass except the Sun.

To understand the demographics of exoplanets

- Precisely identify the types of planets.
  Based on bulk densities (given radius and mass measurements), different possibilities exist for the internal structure.

Madhusudhan et al. 2020

- What planets can harbour life?



Madhusudhan et al. 2023

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Cosmic abundances

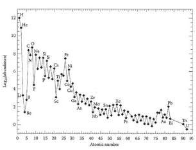
- What's the provenance of planets? What material was it built from?
  Interior models often need some *a priori* knowledge.
- Cosmic abundances are determined by nuclear physics. They can be estimated by observations of the interstellar medium and stars.

Hydrogen dominates, it's the lightest atom.

Helium can be formed in the Big Bang era.

Heavier elements are formed in stars. **Oxygen**, **Carbon**, **Neon**, **Nitrogen** are favorable. **Silicium**.

Iron is the « endpoint » since more massive nuclei are less stable.

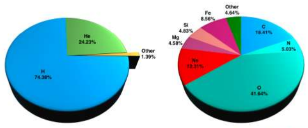


But cosmic abundances evolve with space and time.

Solar system abundances

- Solar system abundances are determined from the photosphere of the Sun.
- They correlate well with relative abundances in meteorites (except for volatiles).

Mass fractions:



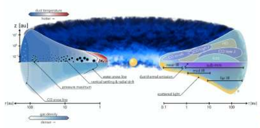
Asplund et al. 2021

Present-day, not exactly protosolar.

The protoplanetary disk as the physical environment

- Elemental abundances are determined by nuclear physics. But most elements are not stable in elemental form but are found as molecules or in compounds.
- Importance of the conditions (Pressure, Temperature) in the protoplanetary disk.

  T can reach a few hundreds degree at 1 AU. That could correspond to P ~ 8 orders of magnitude lower than 1 bar.



Oberg et al. 2021

What can be accreted onto a planet?

- Given the P, T conditions in the disk:

Hydrogen: mostly $\mathbf{H_2}$.

Helium: no chemistry.

Oxygen: $\mathbf{H_2O}$ (not talking about the phase), **CO**

At low P, H_2O condenses as ice around T~170 K.

Water ice line may have been out at around 3-5 AU.

Carbon: $\mathbf{CH_4}$, **CO**, $\mathbf{CO_2}$ (condensing species)

Nitrogen: $\mathbf{N_2}$, $\mathbf{NH_3}$

Mg and Si: $\mathbf{MgSiO_3}$ (enstatite), $\mathbf{Mg_2SiO_4}$ (olivine) (condenses at high T: ~1500K)

Iron: alone or combines with oxides.

What can be accreted onto a planet?

- Most abundant are « gases », then « ices », then « rocks ».

  These are labels of convenience! They do not refer to the phase of the elements.

  Gases: do not condense.

  Ices: can form volatile compounds that condense at low temperatures.

  Rocks: condense at high temperatures.

- **Careful: the form in which elements are added/accreted to planets is not the form they take within the planet!**

  Simple example is hydrogen. Delivered as a gas. But mostly in its metallic form in Jupiter and Saturn.

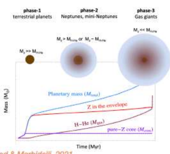
  P, T conditions within the planets are important.

The link between planets and materials in the nebula

- How can planets form? More references: *Helled et al.* 2014, *Draskowska et al.* 2022, …

  Gravitational collapse is not supported by evidence in our solar system.

  Core accretion. Solids (planetesimals, pebbles) aggregating first. *Pollack et al. 1996*



Helled & Morbidelli. 2021

We can interpret our solar system as follows:

Terrestrial planets formed in a region too hot for « ices » to condense. Only silicates and iron formed them. Never massive enough to accrete H_2.

For **gas giants**, accreted solids were rocks and ices. The embryos were massive enough to accrete gas.

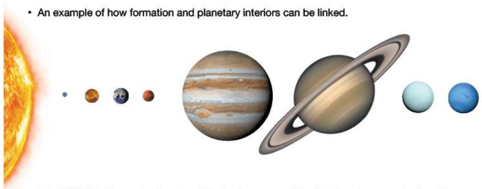
Uranus and Neptune did not accrete a lot of gas (we don't know exactly why, nebula had gone?). Called ice giants but we actually don't know the ice-to-rock ratio.

The presence of ices is not enough. Presence of sufficient mass of solids that matters. Factor 2…

In exoplanetary systems, different populations (hot Jupiters), migration?

The Minimum Mass Solar Nebula

- An example of how formation and planetary interiors can be linked.



- The MMSN is the protoplanetary disk of solar composition that has the amount of metals necessary to build the eight planets of the solar system.

The Minimum Mass Solar Nebula

Solar composition:

Mass fractions of Hydrogen, Helium and Heavy elements, respectively:

$X = 0.734$ $Y = 0.25$ $Z = 0.016$

Planet	Mass	Z
Mercury	$0.06\ M_\oplus$	~1
Venus	$0.82\ M_\oplus$	~1
Earth	$1\ M_\oplus$	~1
Mars	$0.11\ M_\oplus$	~1
Jupiter	$318\ M_\oplus$	~0.05
Saturn	$95\ M_\oplus$	~0.09
Uranus	$15\ M_\oplus$	~0.8
Neptune	$17\ M_\oplus$	~0.8

- Calculate M_z, the total amount of heavy elements in the planets.
- Calculate the amount of hydrogen and helium in the disk for this amount of heavy elements.
- MMSN is of the order of a thousand Earth masses.

The knowledge of Z in the planets can provide information on the protoplanetary disk.

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Why models?

- We cannot go deep inside planets or cut them in half.

  Models are needed to infer indirectly the properties of planetary interiors.

- What do we want to understand?

  We already have an idea of the building blocks and have a rough idea of the overall composition of the different types of planets in our solar system.

- A precise understanding of the internal structure and composition is needed.

Key questions in the solar system

- Do giant planets have a core? How massive?
- What is their heavy-element content? How are they distributed in the interior?
- How much water is there?
- Does the atmospheric composition represent the deep interior?
- What is the temperature structure? Is it adiabatic?
- How do giant planets evolve?
- Is Saturn a smaller version of Jupiter? How different are they?
- How to explain the heat excess of Saturn?
- Are Uranus and Neptune similar?
- What is their ice-to-rock fraction? Are they really « ice » giants?

Key questions in exoplanets

- How to explain the diversity of exoplanets?
- What is the origin of hot-Jupiters?
- What mechanisms explain the inflation of those highly irradiated planets?
- How to interpret the radius valley?
- What is the link between the composition of exoplanets and their host stars?

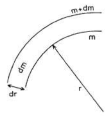
Both fields can help each other!

- The planets in the solar system are our best samples.
- The number of exoplanets may allow us to identify trends.

How to make a model?

- Interior models are often 1D/2D but in reality, planets are 3D.
- They rely on many assumptions (e.g. non-rotating, no magnetic field considered, …)
- An interior model is a solution of the following equations:

i) Mass conservation



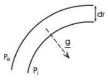
$$\partial m = 4\pi r^2 \rho \partial r$$

$$\boxed{\frac{\partial m}{\partial r} = 4\pi r^2 \rho}$$

(Eulerian form vs Lagrangian form)

How to make a model?

ii) Hydrostatic equilibrium



- The weight of each shell is supported by the pressure difference across the shell.
- The sum of the forces arising from pressure and gravity must be 0:

$$\frac{\partial P}{\partial r} + \rho g = 0$$

$$\boxed{\frac{\partial P}{\partial r} = -\rho g}$$

To counteract gravity, P_i must be larger than P_e. dP/dr is a negative quantity.

- Corrections from rotation later…

How to make a model?

- Simply looking at these 2 equations, we see that we have 3 unknowns.

$$\boxed{\frac{\partial m}{\partial r} = 4\pi r^2 \rho} \quad \boxed{\frac{\partial P}{\partial r} = -\rho g}$$

- This is why we need an **Equation of State (EOS)** to link Pressure and Density.
- Some analytical solutions can be done using barotropes, i.e. rho = rho(P).

Among those, polytropes: $P = K\rho^{1+\frac{1}{n}}$

- In reality, density depends also on temperature.

A well-known case is the ideal gas: $P = \rho k_B T/\mu$

How to make a model?

iii) Energy transport

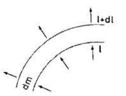
$$\boxed{\frac{\partial T}{\partial r} = \frac{\partial P}{\partial r}\frac{T}{P}\nabla_T} \qquad \nabla_T = \frac{d\ln T}{d\ln P}$$

- We need an EOS that gives rho(P, T).
- ∇_T tells how heat is transported inside the planet.

We need to know the driving mechanisms of heat transport. Convection? Radiation? Conduction?

How to make a model?

iv) Energy conservation



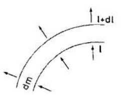
$$\boxed{\frac{\partial L}{\partial r} = 4\pi r^2 \rho \left(\dot{\epsilon} - T\frac{\partial S}{\partial t}\right)}$$

- Luminosity corresponds to the energy loss per unit time.
- Boundary conditions: L = 0 at the center and L = L_{planet} at the surface

- L is the energy entering the shell at the inner surface.
- L+dL is the energy leaving it through the outer surface.
- dL can be provided by cooling, compression, nuclear reactions if it were a star, differentiation, etc…
- A shell can change its internal energy and exchange mechanical work with the neighboring shells.

How to make a model?

iv) Energy conservation



$$\boxed{\frac{\partial L}{\partial r} = 4\pi r^2 \rho \left(\dot{\epsilon} - T\frac{\partial S}{\partial t}\right)}$$

$$dQ = -\frac{dL}{dm}dt$$

dQ is the heat per unit mass added to the shell in the time interval dt

First law of thermodynamics: $dQ = dU + PdV = TdS$

- When a planet forms, material falls and gravitational energy is converted into thermal energy. Planets form hot and then cool. They radiate energy away into space (mostly in the infrared). As they emit radiation, they lose heat.
- But sources of energy can also be : radioactive decay, differentiation (e.g. helium rain), external sources (e.g. tides, impacts?)